Adaptive Control of a 2-DOF Helicopter

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***Abstract*—This paper examines the performance of different control schemes on the 2-DOF Quanser AERO helicopter. We present several control schemes for the single input single output (SISO) system and we analyze the performance of the model reference adaptive control on the coupled multiple input multiple output system (MIMO). We show that for the SISO system, the indirect self-tuning regulator (ISTR) and the MRAC output feedback show a very good performance. Addtionally, the MIMO MRAC controller presents very good model following and good disturbance rejection.**

I. INTRODUCTION

Helicopters are of significant importance due to their extensive usage in military and civilian applications. Helicopters are differentiated by their ability for vertical landing and take-off; however, they often involve complex nonlinear dynamics and unstable systems [1]. Extensive research has been devoted to finding and analyzing different control schemes that guarantee a good performance and stability for helicopters.

Helicopter stability is affected by several factors [2]. First, fuel consumption of the helicopter as well as the variation in the number of passengers, equipment and loading mass affect the weight of the helicopter which in turn affects its system dynamics. Additionally, the deformation of the wings due to the helicopter's aeroelasticity poses further stability concerns. Moreover, as the popularity of unmanned aerial vehicles is increasing, several gadgets are being added to UAVs which further increase the complexity of their systems. These factors, among several others such as severe weather and wind, necessitates the presence of control systems for flying helicopters. While classical control strategies have shown to be effective [3], these issues give rise to adaptive control algorithms which are mainly motivated by their ability to estimate system uncertainties and suppress them. These uncertainties arise due changes in system dynamics, imperfect system modeling, sensor failures among many other reasons. For this reason, we will be using an adaptive controller to account for the change in the dynamics of the 2-DOF helicopter.

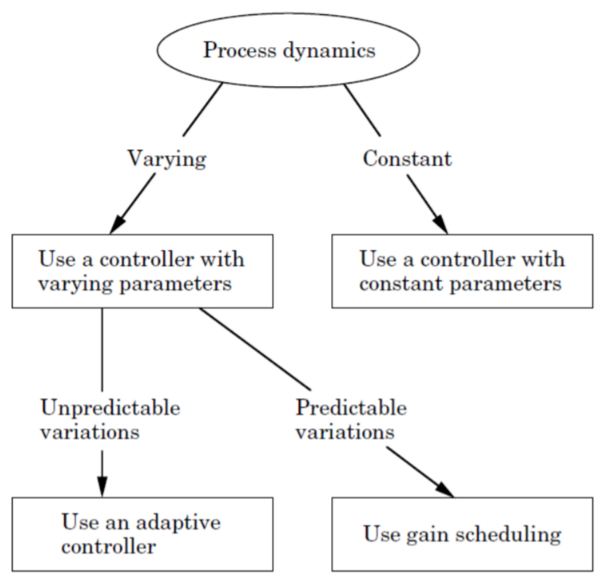

Fig. 1. Motivation behind Controller Choice [4]

In the next section, we will conduct a review of previous research that used a 2 degree of freedom helicopter, specifically the Quanser AERO, as a test bed for control algorithms. We will then focus on previous work done with respect to two control algorithms that we implement in this paper, model reference adaptive control (MRAC) and self-tuning regulator (STR).

II. LITERATURE REVIEW

Using the 2 DOF helicopter, classical control strategies, namely PID, state feedback control (SFC) and Linear Quadratic Regulator (LQR), were designed and tested to compare the performance of each controller [5]. It was determined that while the PID controller achieved desired final value, this was done in a high settling time. On the other hand, SFC was found to pose a limitation of having to compromise between the control signal requirement and the settling time. Finally, LQR was the most effective as it has eliminated this limitation and overcame the drawbacks of PID and SFC. A high performance adaptive augmentation technique applied to an LQR based controller is presented in [6]. The results showed that while the standalone adaptive controller's performance could be improved with further tuning, the adaptive augmentation of the LQR-based controller consistently had a better performance than both the standalone LQR-based controller and the standalone adaptive controller in the presence of unmodeled dynamics and system uncertainties. [7] implements a generic optimal tracking control law for tracking the motion of a 2 DOF helicopter. This method generalizes the implementation of various control methods that can be deployed on a 2 DOF helicopter. In addition, [8] presents a robust nonlinear output feedback control law that achieves attitude tracking of a 2 DOF helicopter. After comparing the system performance to the PID controller, it was shown that the dynamic filter-based controller resulted in a significantly lower steady state error and is capable of compensating for system uncertainties including input-multiplicative parametric uncertainty.

MRAC has proved to be one of the most popular adaptive control schemes which can be based either on Lyapunov's method or the MIT rule [9]. MRAC and deterministic robust control (DRC) were tested on a 2 DOF helicopter [2]. It was found that the two control schemes behave in an opposite manner with regards to transient response and steady state errors. While the robust controller results in fast transient response with poor steady state errors, the adaptive controller results in slower transients with a better asymptotic tracking. Additionally, [10] makes use of an inverse Lyapunov function to present a robust MRAC augmented LQR controller for a 2 DOF helicopter. The authors show that combining MRAC with LQR improves the robustness of the system in response to uncertainties and disturbances, widens the stability region, and yields faster convergence.

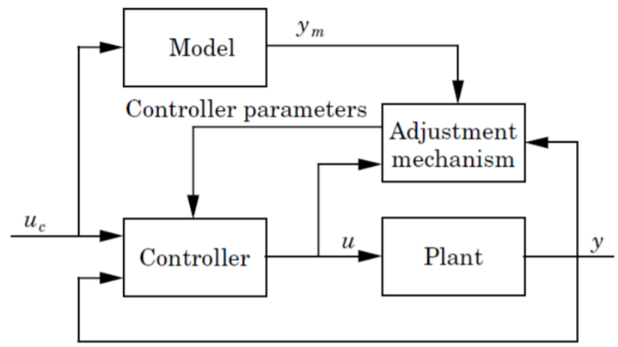

Fig. 2. Block Diagram of a Model Reference Adaptive Controller

Self-tuning regulator (STR) is another adaptive control algorithm, that estimates the parameters of the process transfer function using recursive least square algorithm then updates the controller parameters which is designed via minimum degree pole placement [11]. In this paper, we will be assessing and analyzing the performance of both MRAC and STR when applied to the Quanser AERO 2 DOF helicopter.

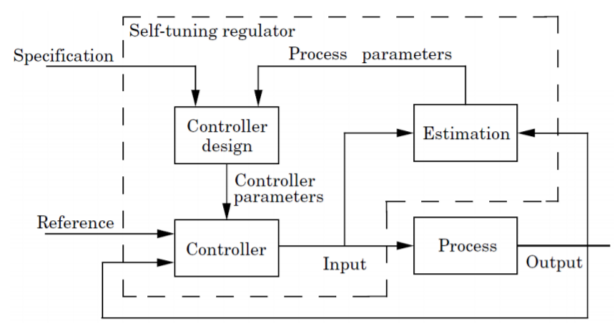

Fig. 3. Block Diagram of a Self-tuning Regulator [4]

The rest of this paper is organized as follows; section III presents the dynamic model of the system. Section IV presents the controller design for the Quanser AERO 2 DOF helicopter. Section V discusses the simulation results. We finally conclude our analysis in section VI.

III. DYNAMIC MODEL

The Quanser Aero consists of two propellers, one for the pitch and one for the yaw. We will be representing the pitch as θ and the yaw as ψ.The yaw propeller is perpendicular to the grounds and the pitch propeller is perpendicular to it. The zero level for the pitch angle θ=0 is when the propeller is parallel to the ground [12].

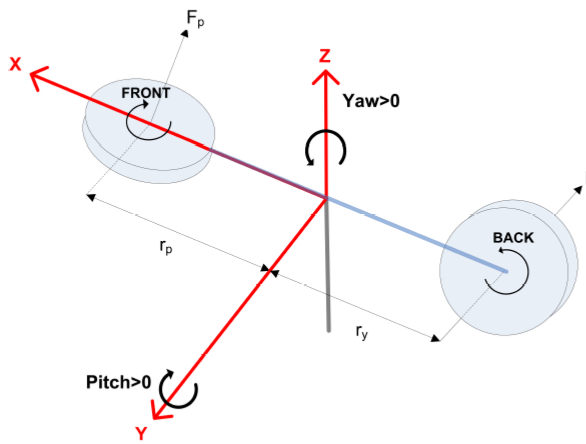

Fig. 4. Free body diagram of the Quanser Aero

The forces that act on each axis are the inertial force, the damping, the stiffness, and the torque applied by the propellers. Given this, the equations of motion are:

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp} = \tau_p$$

$$J_y\ddot{\psi} + D_y\dot{\psi} = \tau_y$$

Each propeller creates a torque on both axes. The pitch propeller causes a force perpendicular to the pitch axis by creating a force F_p shown in [**?**], perpendicular to the pitch axis. It also causes a torque along the yaw axis due to the rotation. Given this, the equations of the torques are:

$$\tau_p = K_{pp}V_p + K_{py}V_y$$

$$\tau_y = K_{yp}V_p + K_{yy}V_y$$

Substituting the value of the torques in the equations of motions we get the following differential equations:

$$J_p\ddot{\theta} + D_p\dot{\theta} + K_{sp} = K_{pp}V_p + K_{py}V_y$$

$$J_y\ddot{\psi} + D_y\dot{\psi} = K_{yp}V_p + K_{yy}V_y$$

Taking the Laplace transform for the equations on motion and assuming zero initial conditions

$$\theta(J_p s^2 + D_p s + K_{sp}) = K_{pp}V_p + K_{py}V_y$$

$$\psi(J_y s^2 + D_y s) = K_{yp}V_p + K_{yy}V_y$$

This MIMO system can either be represented using four transfer functions or in state-space formulation. The transfer functions are as follows:

$$\frac{\theta}{V_p} = \frac{K_{pp}}{(J_p s^2 + D_p s + K_{sp})} \quad (1)$$

$$\frac{\theta}{V_y} = \frac{K_{py}}{(J_p s^2 + D_p s + K_{sp})} \quad (2)$$

$$\frac{\psi}{V_p} = \frac{K_{py}}{(J_y s^2 + D_y s)} \quad (3)$$

$$\frac{\psi}{V_y} = \frac{K_{yy}}{(J_y s^2 + D_y s)} \quad (4)$$

The in state-space formulation, the equations can be written as $\dot{x} = Ax + Bu$ and $y = Cx + Du$ where

$$x^T = \begin{bmatrix} \theta & \psi & \dot{\theta} & \dot{\psi} \end{bmatrix}$$

$$y^T = \begin{bmatrix} \theta & \psi \end{bmatrix}$$

$$u^T = \begin{bmatrix} V_p & V_y \end{bmatrix}$$

And the A, B, C, and D matrices are:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{sp}}{J_p} & 0 & -\frac{D_p}{J_p} & 0 \\ 0 & 0 & 0 & -\frac{D_y}{J_y} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_p} & \frac{K_{py}}{J_p} \\ \frac{K_{yp}}{J_y} & \frac{K_{yy}}{J_y} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

IV. CONTROLLER DESIGN

For the controller design, we decided to dive into two major categories. First, we decided to study the model as a decoupled SISO system, studying the behaviour and effect of each transfer function on the desired outputs θ and ψ. The main reason behind this category is to have a better look at the impact of each input on the outputs. Real life applications for this decoupled system could be seen when a 2-DOF helicopter has a damaged motor and thus making it unusable by the operator. In this case it would be optimal to turn it off and use the second motor for operation. For the second category, we opted for a full state Model Reference Adaptive Controller for the coupled MIMO system since we have seen many times the good performance of this type of controller and since it would be appropriate for a MIMO system unlike other controllers such as a standard Self Tuning Regulator.

A. Single Output Single Input (SISO)

In this section, we will present the design of the following controllers used for each SISO transfer function of the MIMO system: Normalized MIT rule, Indirect Self Tuning Regulator with disturbance rejection, Full State Model Reference Adaptive Control and finally Output Feedback Model Reference Adaptive Control.

1) SISO MRAC Full-State Feedback: The model reference adaptive controller is an adaptive system that forces the output (x) to a suggested reference model (x_m). It measures the error between the output and the reference model for all the states and a regressor adapts the parameters of the control law to zero. Consider the reference model to be a decoupled system where V_p only affects the pitch and V_y only affects the yaw. Assume the transfer functions to be second order with a natural frequency w_n of 15 and a damping ration ζ of 1, corresponding to 0 overshoot and a a settling time of 0.2 seconds. In state-space formulation the reference model is written as

$$\dot{x_m} = A_m x_m + B_m u_c$$

where

$$A_m = \begin{bmatrix} 0 & 1 \\ -w_n^2 & -2\zeta w_n \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 \\ w_n^2 \end{bmatrix}$$

We suggest a controller structure with two degrees of freedom, a feedforward and a feedback term, of the form:

$$u(t) = Mu_c(t) - Lx(t)$$

where:

$$M = M_1$$

$$L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}$$

This results in a closed loop system described by:

$$\dot{x} = (A - Bl)x + BMu_c$$

Which can be written as:

$$\dot{x} = A_c x + B_c u_c$$

To guarantee model following we want A_c and B_c to reach A_m and B_m respectively. Solving for the true values of M, and L we get:

$$\Theta^{0T} = \begin{bmatrix} M_1^0 & L_1^0 & L_2^0 \end{bmatrix}$$

where:

$$M_1^0 = \frac{w_n^2 J_p}{K_{pp}}$$

$$L_1^0 = \frac{(J_p w_n^2 - K_{sp})}{K_{pp}}$$

$$L_2^0 = \frac{(2\zeta w_n J_p - D_p)}{K_{pp}}$$

The error in the system is the difference between the full-state output and the reference model:

$$e = x - x_m$$

$$\dot{e} = \dot{x} - \dot{x_m}$$

$$\dot{e} = -A_m e + \Psi(\Theta - \Theta^0) = -A_m e + \Psi\tilde{\Theta}$$

where the regressor is:

$$\Psi = \begin{bmatrix} 0 & u_c \\ 0 & \theta \\ 0 & \dot{\theta} \end{bmatrix}$$

The suggested Lyapunov function is:

$$V(e, \Theta) = \frac{1}{2}(e^T Pe + \tilde{\Theta^T}\Gamma^{-1}\tilde{\Theta})$$

Where P is given by $A_m^T P + PA_m = -Q$ and Q is a 2x2 identity matrix, and Γ is a 3x3 diagonal adaptation matrix.

$$\dot{V} = -\frac{1}{2}e^T Qe + \tilde{\Theta}^T(\Gamma^{-1}\dot{\tilde{\Theta}} + \Psi^T Pe)$$

Finally the adaptation law is chosen as

$$\dot{\tilde{\Theta}} = -\Gamma\Psi^T Pe$$

The coefficients of the regressor Ψ are unknown so they were absorbed by the adaptation matrix, taking into account their sign, which is considered to be known.

The same process can be done for the other relations between the inputs and outputs.

2) Normalized MIT rule: We decided to use the same reference model and controller design (two degrees of freedom) as the MRAC full-state feedback. The difference is in the adaptation law for the parameters. While the MRAC uses stability theory, the MIT rules uses the gradient method. The controller structure, regressor, and parameters used are as before:

$$u(t) = Mu_c(t) - Lx(t)$$

$$\phi = \begin{bmatrix} 0 & u_c \\ 0 & \theta \\ 0 & \dot{\theta} \end{bmatrix}$$

$$\Theta = \begin{bmatrix} M_1 & L_1 & L_2 \end{bmatrix}$$

These parameters are updated using the normalized MIT rule with an $\alpha = 0.01$

$$\frac{d\Theta}{dt} = \frac{\gamma\phi e}{\alpha + \phi^T\phi}$$

The same process can be done for the other relations between the inputs and outputs.

3) Indirect Self Tuning Regulator with disturbance rejection: As a proof of derivation, we will use the transfer function relating θ to V_p in equation (1). Since the transfer function is second order, the Recursive Least Square model used is:

$$\phi(t) = \begin{bmatrix} -sy_f & -y_f & su_f & u_f \end{bmatrix}$$

$$\Theta = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \end{bmatrix}$$

Where $a_1 = \frac{D_p}{J_p}$, $a_2 = \frac{K_{sp}}{J_p}$, $b_1 = 0$ and $b_2 = \frac{K_{pp}}{J_p}$. We are also using filtered versions of the input and output since we are taking their derivatives which is undesirable and can cause instabilities. The filtered versions are computed by implementing a pre-filter H_f such that:

$$H_f = \frac{1}{A_m} = \frac{1}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$$y_f = \frac{1}{H_f}y$$

$$u_f = \frac{1}{H_f}u$$

The following parameter update law is used:

$$\frac{d\hat{\Theta}}{dt} = P(t)\phi(t)(s^2 y_f(t) - \phi^T(t)\hat{\Theta}(t)$$

$$\frac{dP(t)}{dt} = \alpha P(t) - P(t)\phi^T(t)\phi(t)P(t)$$

We chose a standard 2-DOF control law $R(s)u(t) = T(s)u_c(t) - S(s)y(t)$ which includes both feed-forward and feedback terms for proper model following.Abiding by the minimum degree pole placement criteria, the polynomials $R(s), S(s)$ and $T(s)$ as well as the observer polynomial $A_o(s)$ are all first order polynomials where $R(s)$ and $A_o(s)$ are monic. Solving Diophantine's equation $A(s)R(s) + B(s)S(s) = A_o(s)A_m(s)$ and setting $T(s) = B_m'(s)A_o(s)$, we obtain the following parameter dependencies:

$$R(s) = s + r_1 = s + (2\zeta\omega_n + a_o - a_1)$$

$$S(s) = s_0 s + s_1$$

$$= \frac{\omega^2 + 2\zeta\omega_n a_o - a_2 - a_1 r_1}{b_2}s + \frac{a_o\omega^2 - a_2 r_1}{b_2}$$

$$T(s) = \frac{\omega^2(s + a_o)}{b_2}$$

4) Output Feedback Model Reference Adaptive Control: Taking a look at the high-fidelity model transfer functions in equations (1), (2), (3) and (4), we notice that all of the all them have a relative degree of two which implies that they are not Strictly Positive Real. Therefore, we opted for a modified output feedback Model Reference Adaptive Controller that where $G(s)$ is not SPR with a relative degree of two and where $b0$ is unknown but with known sign. In an output feedback design the system adapts to the reference model. The control law and the regressor to update the parameters are obtained from the Diophantine equation. Again, we will only consider the transfer function between the pitch and V_p; the other relations are derived similarly. The transfer function has a relative degree of 2 so it is not SPR. This can be written as

$$Ay(t) = b_0Bu(t)$$

Where B=1 and b_0 is considered to be unknown. Next, we solve the Diophantine equation. Since the plant and the reference model are the same as the ones used in the ISTR design we obtain the same values for r_1, s_0, s_1, t_0, t_1. The linear controller we will be considering will be:

$$A_0A_m = AR_1 + b_0S$$

And T will be chosen as $T = t_0s + t_1$

Next we derive the error dynamics of the control structure, which is the difference between the plant output and the reference model output.

$$e = y - y_m$$

$$e = \frac{b_0}{A_0A_m}(Ru + Sy - Tu_c)$$

Since the transfer function is not SPR we must introduce a filtered error e_f

$$e_f = \frac{Q}{P}e$$

The degree of Q must be at least the degree of A_0A_m, so we choose $Q = A_0A_m$, $P = P_1P_2$, $P_1 = A_m$, $P_2 = A_0$

$$e_f = b_0(\frac{R}{P}u + \frac{S}{P}y - \frac{T}{P}u_c)$$

$$e_f = b_0(\frac{1}{P_1}u + \frac{R - P_2}{P}u + \frac{S}{P}y - \frac{T}{P}u_c)$$

$$e_f = b_0(\frac{1}{P_1}u + \frac{R_1}{P}u + \frac{S}{P}y - \frac{T}{P}u_c)$$

Where $R_1 = R - P$ and $r_1{}'$ is the coefficient of the polynomial. This is the only difference between the parameters in the case of output feedback and the case of ISTR. The resulting parameters vector is:

$$\Theta = \begin{bmatrix} r_1{}' & s_0 & s_1 & t_0 & t_1 \end{bmatrix}^T$$

And the resulting regressor will be

$$\phi^T = \frac{1}{T}\begin{bmatrix} u & sy & y & -su_c & -u_c \end{bmatrix}^T$$

Since the value of b_0 used in the control law is unknown, we would need to use an estimate instead. First we define the augmented error:

$$\varepsilon = e_f + \hat{b}_0\eta$$

where

$$\eta = -(\frac{1}{P_1}u(t) + \phi^T\Theta(t))$$

Then the adaptations law for $b0$ and Θ are

$$\dot{\hat{b}}_0 = -\gamma_1\eta\varepsilon$$

$$\dot{\hat{\Theta}} = -\gamma_2' sign(b_0)\phi\varepsilon$$

Finally the control law is given by:

$$u(t) = -\Theta^T(t)[P_1\phi(t)]$$

B. Multiple Output Multiple Input (MIMO) MRAC Full-State Feedback

The model reference adaptive controller is an adaptive system that forces the output (x) to a suggested reference model (x_m). It measures the error between the output and the reference model for all the states and a regressor adapts the parameters of the control law to zero. Consider the reference model to be a decoupled system where V_p only affects the pitch and V_y only affects the yaw. Assume the transfer functions to be second order with a natural frequency w_n of 15 and a damping ration ζ

of 1, corresponding to 0% overshoot and a settling time of 0.2 seconds. In state-space formulation the reference model is written as:

$$\dot{x}_m = A_m x_m + B_m u_c$$

where:

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\omega_n^2 & 0 & -2\zeta\omega_n & 0 \\ 0 & -\omega_n^2 & 0 & -2\zeta\omega_n \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \omega_n^2 & 0 \\ 0 & \omega_n^2 \end{bmatrix}$$

We suggest a controller structure with two degrees of freedom, a feed-forward and a feedback term, of the form:

$$u(t) = Mu_c(t) - Lx(t)$$

where:

$$M = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix}$$

$$L = \begin{bmatrix} L_1 & L_2 & L_3 & L_4 \\ L_5 & L_6 & L_7 & L_8 \end{bmatrix}$$

This results in a closed loop system described by:

$$\dot{x} = (A - Bl)x + BMu_c$$

Which can be written as:

$$\dot{x} = A_c x + B_c u_c$$

To guarantee model following we want A_c and B_c to reach A_m and B_m respectively. Solving for the true values of M, and L we get:

$$M_1 = \frac{\omega_n^2 J_p K_{yy}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$M_2 = \frac{\omega_n^2 J_y K_{py}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

$$M_3 = \frac{-\omega_n^2 J_p K_{yp}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$M_4 = \frac{-\omega_n^2 J_y K_{pp}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

$$L_1 = \frac{(J_p\omega_n^2 - K_{sp})K_{yy}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$L_2 = \frac{\omega_n^2 J_y K_{py}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

$$L_3 = \frac{(2\zeta\omega_n J_p - D_p)K_{yy}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$L_4 = \frac{(2\zeta\omega_n J_y - D_y)K_{py}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

$$L_5 = \frac{(J_p\omega_n^2 - K_{sp})K_{yp}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$L_6 = \frac{\omega_n^2 J_y K_{pp}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

$$L_7 = \frac{(2\zeta\omega_n J_p - D_p)K_{yp}}{K_{pp}K_{yy} - K_{py}K_{yp}}$$

$$L_8 = \frac{(2\zeta\omega_n J_y - D_y)K_{pp}}{K_{py}K_{yp} - K_{pp}K_{yy}}$$

The error in the system is the difference between the full-state output and the reference model:

$$e = x - x_m$$

$$\dot{e} = \dot{x} - \dot{x_m}$$

$$\dot{e} = -A_m e + \Psi(\Theta - \Theta^0) = -A_m e + \Psi\tilde{\Theta}$$

Where the regressor is 4 by 12 matrix:

$$\Psi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & & & & & & & & \\ u_{cp} & u_{cy} & u_{cp} & u_{cy} & \theta & \psi & \dot{\theta} & \dot{\psi} & \theta & \psi \\ \dot{\theta} & \dot{\psi} & & & & & & & & \\ u_{cp} & u_{cy} & u_{cp} & u_{cy} & \theta & \psi & \dot{\theta} & \dot{\psi} & \theta & \psi \\ \dot{\theta} & \dot{\psi} & & & & & & & & \end{bmatrix}$$

The suggested Lyapunov function is

$$V(e, \Theta) = \frac{1}{2}(e^T P e + \tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta})$$

Where P is given by $A_m^T P + PA_m = -Q$ and Q is a 4x4 identity matrix, and Γ is a 12x12 diagonal adaptation matrix.

$$\dot{V} = -\frac{1}{2}e^T Q e + \tilde{\Theta}^T(\Gamma^{-1}\dot{\tilde{\Theta}} + \Psi^T P e)$$

Finally the adaptation law is chosen as

$$\dot{\tilde{\Theta}} = -\Gamma \Psi^T P e$$

The coefficients of the regressor Ψ are unknown so they were absorbed by the adaptation matrix, taking into account their sign, which is considered to be known.

V. SIMULATION RESULTS

The Matlab and Simulink simulations were obtained for a square wave command input u_c whose amplitude is $40°$, period is $50\ s$, voltage limit of $V_{limit} = 24V$, simulation time of $200\ s$ and a sampling frequency of $2kHz$.

A. SISO results

In this section, we will present the results obtained for the SISO controller used for the models depicted in (1), (2), (3) and (4).

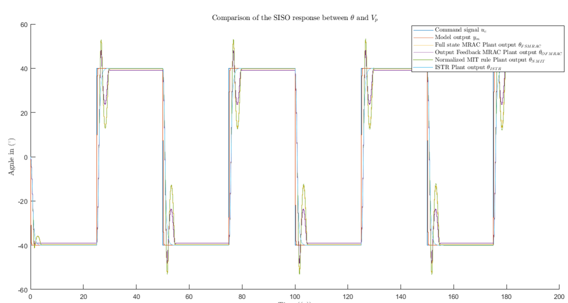

Fig. 5. Comparison of the SISO response between θ and V_p

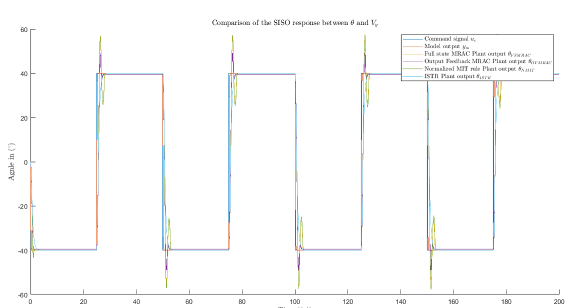

Fig. 6. Comparison of the SISO response between θ and V_y

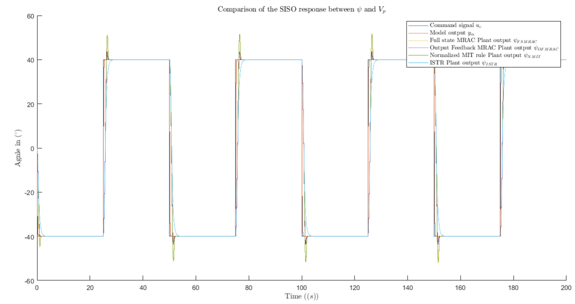

Fig. 7. Comparison of the SISO response between ψ and V_p

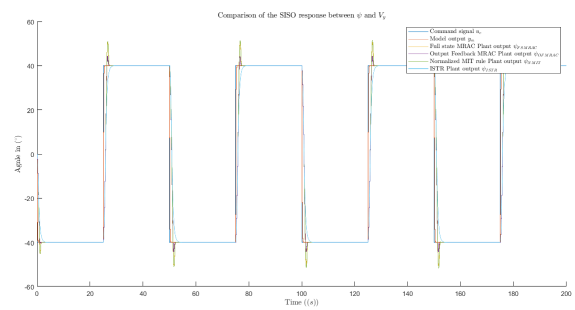

Fig. 8. Comparison of the SISO response between ψ and V_y

For all the SISO controller designs, we notice the same trend over all the systems. All the outputs eventually follow the reference model and reach zero steady state error. The difference between them is in the transient phase. We notice that the normalized MIT design and the full-state feedback MRAC have almost exact responses. This is very expected as they share the controller structure and observer polynomials; they only differ in the adaptation laws. For both these designs we chose low values for the adaptation gain. Since the transfer functions are not SPR these systems do not perform well at high gains. For the output feedback controller, we notice a similar behavior with a smaller overshoot. The overshoot persists since the parameters still did not converge. Finally, the ISTR controller showed the best results compared to the other controller designs. It had no overshoot and a much better settling time.

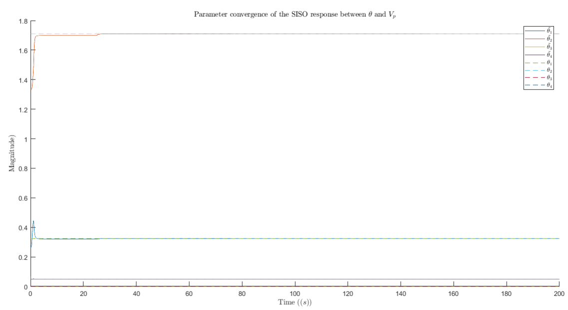

Fig. 9. Parameter convergence of the $\frac{\theta}{V_p}$ model using an ISTR

As shown in figure 9, the parameters in the ISTR model quickly converge. This is because we were able to put a large adaptation gain.

Figure 10 shows the power spectrum of the input u for the normalized MIT rule. The power spectrum of the inputs of the Full state MRAC controller as well as the Output feedback MRAC controller highly ressemble the plotted input and were therefore ommitted. It can be seen that there is one major peak at the origin and thus the input does not share an equal power over all frequencies. Therefore, the input is low PE and parameter convergence cannot be achieved using this controller input.

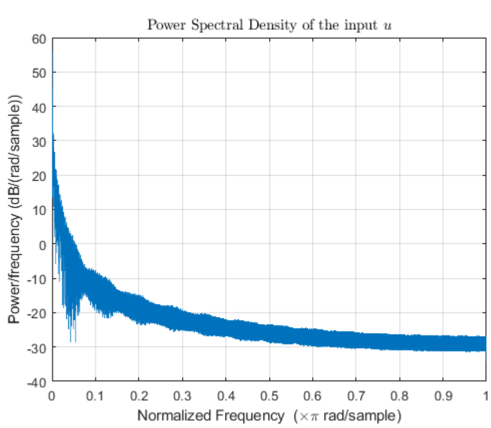

Fig. 10. Power Spectrum of the input u

B. MIMO results

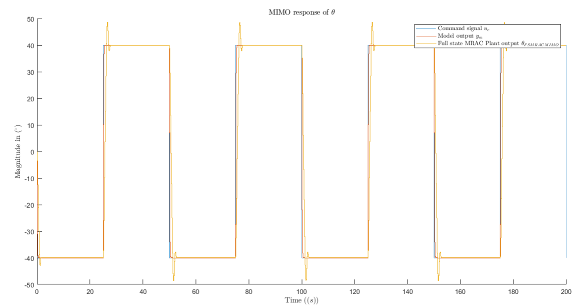

Fig. 11. MIMO response of θ

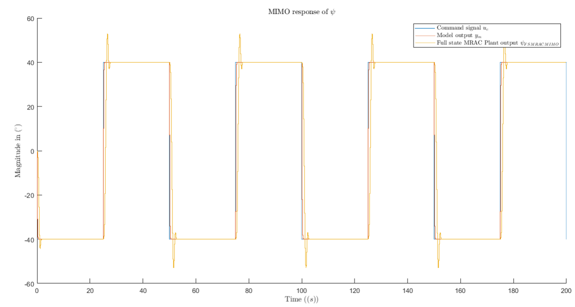

Fig. 12. MIMO response of ψ

As shown in figures 11 and 12, the MIMO response of both θ and ψ under full state MRAC shows very good results. Both figures show fast tracking and good transient response. Furthermore, no steady state error is shown in both responses and we finally note that while both the pitch and yaw response have a relatively small overshoot, the overshoot for the response of the yaw is slightly larger than that of the pitch.

C. Input Step Disturbance

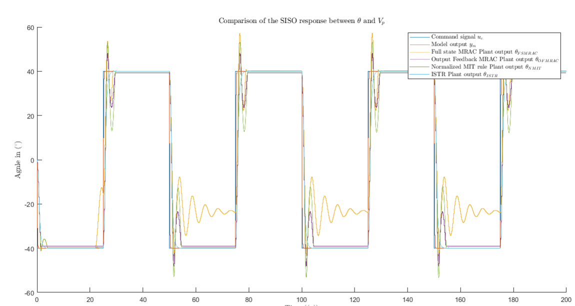

Fig. 13. θ vs V_p input disturbance response

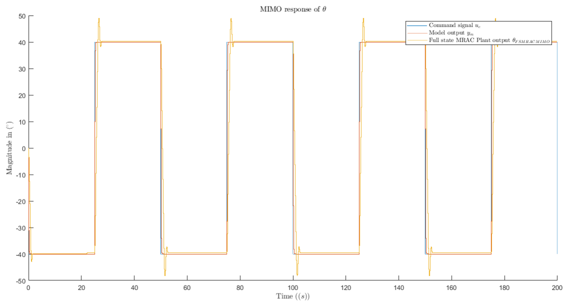

Fig. 14. MIMO input disturbance response of θ

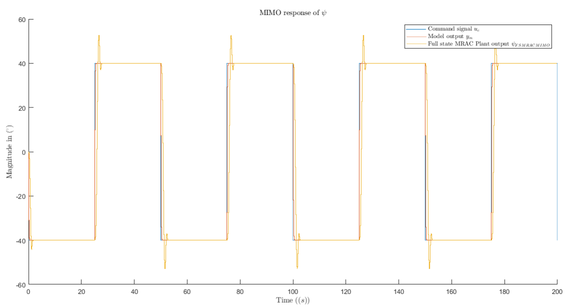

Fig. 15. MIMO input disturbance response of ψ

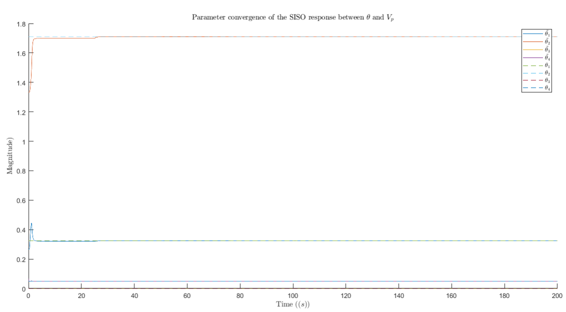

Fig. 16. Parameter convergence of ISTR response of $\theta\ vs\ V_p$ for an input disturbance

We tested the SISO controller structures' ability to reject a step input. We added a step of $10V$ at $22\ s$ at the input of the plant and figure 13, 14 and 15 shows the output results. We notice that most the controllers (other than the ISTR) showed an increase in the overshoot and an overall deterioration in the output. The full-state MRAC showed a very large steady state error. This is expected as nothing in the design included disturbance rejection. In the ISTR, we accounted for a step input disturbance in the model by a dding a derivative component to the closed loop system since we know the dynamics of a step disturbance. So, the high adaptation gain allowed the system to quickly account for the disturbance and reject it as shown in figure 16 where other models with a lower adaptation gain failed.

D. Output Disturbance Rejection

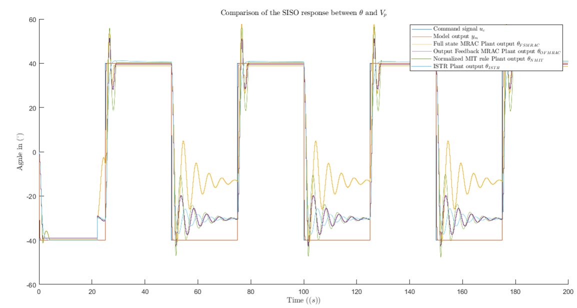

Fig. 17. $\theta\ vs\ V_p$ ouptut disturbance response

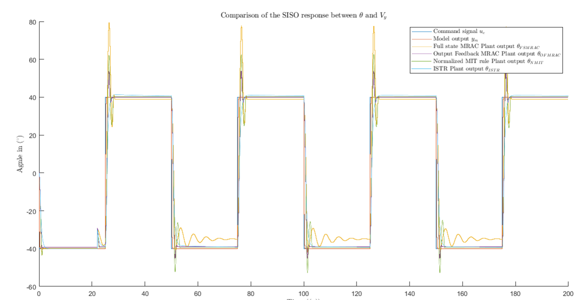

Fig. 18. $\theta\ vs\ V_y$ output disturbance response

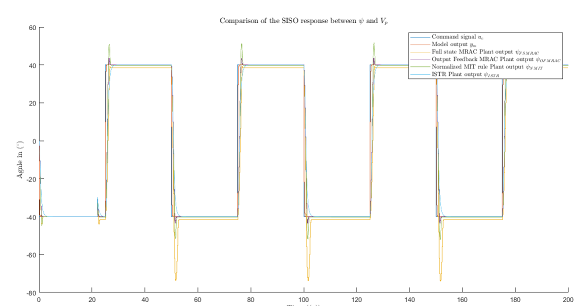

Fig. 19. $\psi\ vs\ V_p$ output disturbance response

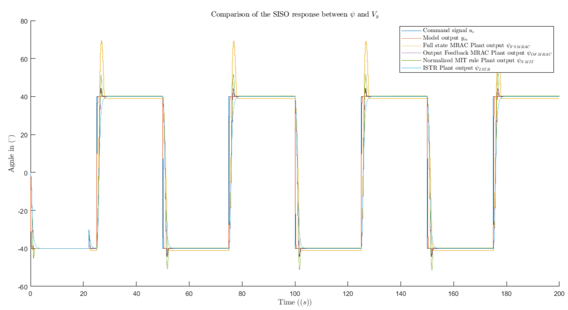

Fig. 20. ψ vs V_y output disturbance response

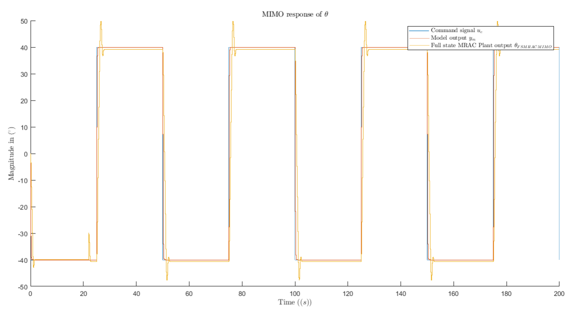

Fig. 21. θ MIMO output disturbance response

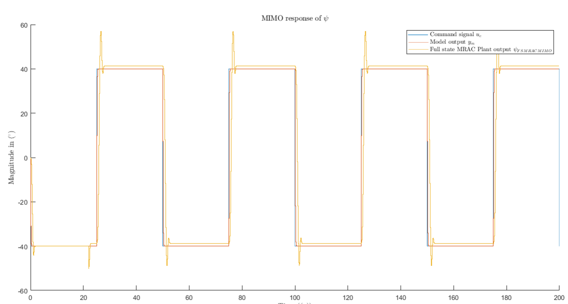

Fig. 22. ψ MIMO output disturbance response

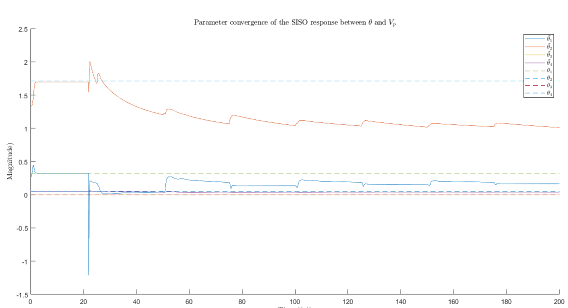

Fig. 23. Parameter convergence of ISTR response of θ vs V_p for an output disturbance

We also tested the controllers ability to reject a step output disturbance. We added a disturbance of $10°$ at 22 s at the output of the plant. From figure 17, we can see that there is significant output steady state error for all the SISO controllers and this is due to the lack of convergence of the parameter estimates. In figure 18,the MRAC full state controller has the most overshoot while the ISTR controllers achieves relatively good model output tracking. From figures 19 and 20, the ψ repsonse is very good for all controllers excpet for the full state MRAC controller which experiences high overshoot. The MIMO results presented in figures 21 and 22 show good output tracking with a small steady state error for both responses. The behaviour in figure 17 of the ISTR is explained by the lack of parameter convergence shown in figure 23.

VI. CONCLUSION

SISO and MIMO adaptive schemes were presented for a 2-DOF Quanser AERO helicopter in this paper. Simulations of the models and the controllers were performed on MATLAB and SimuLink. As shown in the simulations, the MIMO response under the full state MRAC adaptive scheme showed very good results in terms of steady state error, tracking and transient response as well as good disturbance rejection. Additionally, for the SISO control schemes, the ISTR provided the best results in both the transient and the steady state repsonses. Furthermore, the ISTR model had good parameter convergence and did not show any overshoot when there was no output disturbance. Future work may include an in depth analysis of the parameter convergence behaviour of the presented controllers in order to optimize the model following behaviour as well as the disturbance rejection capabilities of the controller.

VII. ACKNOWLEDGMENT

We would like to thank Professor Naseem Daher for his constant support throughout the semester and over the course of this project.

VIII. GLOSSARY

TABLE I
SYMBOL DEFINITION

Symbol	Parameter
J_p	Total moment of inertia about the pitch axis
J_y	Total moment of inertia about the yaw axis
D_p	Damping about the pitch axis
D_y	Damping about the yaw axis
K_{sp}	Stiffness about the pitch axis
K_{pp}	Torque thrust gain from pitch motor
K_{yy}	Torque thrust gain from yaw motor
K_{py}	Cross-torque thrust gain acting on the pitch from the yaw motor
K_{yp}	Cross-torque thrust gain acting on the yaw from the pitch motor
V_p	The voltage applied on the pitch motor
V_y	The voltage applied on the yaw motor
τ_p	Torque applied on the pitch axis
τ_y	Torque applied on the yaw axis

TABLE II
SYMBOL VALUES

Symbol	Parameter
J_p	0.0219 Kgm^2
J_y	0.0220 Kgm^2
D_p	0.0071 Nms/rad
D_y	0.0220 Nms/rad
K_{sp}	0.0375 Nms/rad
K_{pp}	0.0011 Nm/V
K_{yy}	0.0022 Nm/V
K_{py}	0.0021 Nm/V
K_{yp}	-0.0027 Nm/V

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